How to proof the Collatz conjecture - an approach by reversing the sequences

Explanation for up and down and 4 rules for reversing Collatz sequences

The behaviour of up and down of a Collatz sequence can be explained by using the binary system and n+(n+1)/2 instead of (3n+1)/2

Example 255 (with mod 4 = 3):

```
       n =   11111111 (255) mod 4 = 3
+(n+1)/2 =    1000000 (128) number to add
           __________
  Result =  101111111 (383) mod 4 = 3
```

Example 27 (mod 4 = 3):

```
       n =   11011 (27) mod 4 = 3
+(n+1)/2 =    1110 (14) number to add
           __________
  Result =  101001 (41) mod 4 = 1
```

Next iteration (with 41 mod 4 =1):

```
       n =  101001 (41) mod 4 = 1
+(n+1)/2 =   10101 (21) number to add
           __________
  Result =  111110 (62) mod 4 = 1 -> halving: 11111 (31) mod 4 = 3
```

We can say:

- n mod 4 = 3 let the net result (the number divided by 2 until it is odd) grow, until the iteration reach a number with mod 4 = 1
- n mod 4 = 1 let the net result (the number divided by 2 until it is odd) shrink, until the iteration reach a number with mod 4 = 3 (or the number is 1)

Additionally, there are 4 rules for reversing the Collatz sequences:

1. Any number n, which leads with the Collatz-rules to the end loop 4-2-1, multiplied by 2^x (x from 1 to ∞), will also lead to 4-2-1; the result will be even and you can either repeat this instruction or, if the result’s mod 6 = 4, you can subtract 1 and divide its result by 3 for a new number (which will be odd, thus, this instruction or instruction 2 can be applied, or depending on the result‘s mod 3, instruction 3 or 4)
2. Any odd number n, which leads with the Collatz-rules to the end loop 4-2-1, multiplied by 4 and then added 1, will also lead to 4-2-1; and because the result is again an odd number, this instruction or instruction 1 can be applied, or depending on the result‘s mod 3, instruction 3 or 4
3. Any odd number n with n mod 3 = 2, which leads with the Collatz-rules to the end loop 4-2-1, subtracted (n+1)/3, will also lead to 4-2-1; and because the result is again an odd number, instruction 1 or 2 can be applied or, depending on the result‘s mod 3, this instruction or instruction 4
4. Any odd number n with n mod 3 = 1, which leads with the Collatz-rules to the end loop 4-2-1, added (n-1)/3, will also lead to 4-2-1; and because the result is again an odd number, instruction 1 or 2 can be applied or, depending on the result‘s mod 3, this instruction or instruction 3

Source: How to proof the Collatz conjecture - an approach by reversing the sequences

Written by Joerg Drescher, Kyiv/Berlin, March.2022